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9. Are the groups $\mathbb{Z}_8$ and $\mathbb{Z}_2 \times \mathbb{Z}_4$ isomorphic? (The operation in each of the groups $\mathbb{Z}_8$, $\mathbb{Z}_2$, and $\mathbb{Z}_4$ is addition.) Explain your answer.

NO. $\mathbb{Z}_8$ has an element (1) of order 8; but every element in $\mathbb{Z}_2 \times \mathbb{Z}_4$ has order less than or equal to 4.

10. Which of the groups $(\mathbb{C} \setminus \{0\}, \times)$, $(\mathbb{R}, +)$, $(\mathbb{R} \setminus \{0\}, \times)$, and $(\mathbb{R}^{\text{pos}}, \times)$ are isomorphic? (Be sure to examine each pair of groups. I use $\mathbb{R}^{\text{pos}}$ to represent the set of positive real numbers.) Explain your answer.

$\mathbb{C} \setminus \{0\}$ is not isomorphic to any of the other groups because it contains i which has order 4. $\mathbb{R}$ does not contain any elements of order 4 because the only solution of $4x = 0$ is $x = 0$. $(\mathbb{R} \setminus \{0\}, \times)$ and $(\mathbb{R}^{pos}, \times)$ do not contain any elements of order 4 because the only real solutions of $x^4 = 1$ are ± 1 and these numbers have order 2 or less.

The function $\varphi : (\mathbb{R}, +) \to (\mathbb{R}^{pos}, \times)$ given by $\varphi(x) = 2^x$ is a group isomorphism as was shown in class.

$(\mathbb{R}, +)$ is not isomorphic to $(\mathbb{R} - \{0\}, \times)$ because -1 is an element of order 2 in $\mathbb{R} \setminus \{0\}$, but $(\mathbb{R}, +)$ does not contain any elements of order 2 because the only solution of $2x = 0$ is $x = 0$.

The only isomorphism is $(\mathbb{R}, +) \simeq (\mathbb{R}^{pos}, \times)$