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7. Let H be the subgroup $\{(1), (12), (13), (23), (123), (132)\}$ of S_4. Let x be the element (124) of S_4, and let $[x] = \{y \in G \mid xy^{-1} \in H\}$. List the elements of $[x]$. (Each element of $[x]$ should appear in your list exactly once.)

The elements of $[X]$ are

$(1)(124) = (124)$
$(12)(124) = (24)$
$(13)(124) = (1243)$
$(23)(124) = (1324)$
$(1231)(124) = (13)(24)$
$(132)(124) = (243)$

8. How many permutations in S_6 have order 3? Explain your answer.

S_6 has $2 \cdot \binom{6}{3}$ 3-cycles

S_6 has $5 \cdot 40 \cdot 2$ elements of the form $(abc)(def)$

because we may as well put 1 first $(1\_\_)(\_\_\_)$

5 choices · 4 choices · 2 choices · 1 choice

Put the smallest remaining number here

S_6 has $40 + 40 = 80$ elements of order 3