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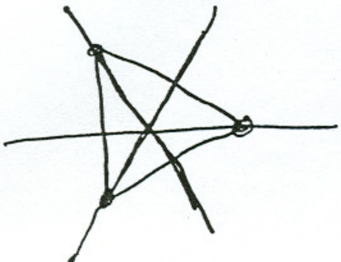
7. True or False (If true, then prove it. If false, then give a counterexample.) If every proper subgroup of the group G is abelian, then G is abelian. (Recall that the subgroup H is G is a *proper* subgroup if $H \neq G$.)

False The easiest counter example is D_3 which has six elements $id, \rho, \rho^2, \sigma, \sigma\rho, \sigma\rho^2$

where ρ is rotation by 120°, σ is reflection across the x-axis



$\rho\sigma = \sigma\rho^2$ so D_3 is not abelian

but every proper subgroup of D_3 has order 1, 2, or 3 by Lagrange's Theorem and every group of order 1, 2, or 3 is cyclic hence abelian.