

7. True or False (If true, then prove it. If false, then give a counterexample.) If $(G, *)$ is an abelian group and $H = \{g \in G \mid g * g = e\}$, then H is a subgroup of G .

True: closure if g and $h \in H$, then $(gh)gh = gghh = ee = e$
so $gh \in H$

Identity $ee = e$ so $e \in H$

Assoc $*$ associates on G so $*$ associates on the subset H

Inverses If $g \in H$ then $gg = e$ so (multiply by g^{-1} twice)

$e = g^{-1}g$ Thus $g^{-1} \in H$.

so H is a subgroup of G

8. True or False (If true, then prove it. If false, then give a counterexample.) If $(G, *)$ is a group and $H = \{g \in G \mid g * g = e\}$, then H is a subgroup of G .

False Here is an example. Let $G = D_4$. Observe that σ and $\sigma\rho$ are both in H (because $\sigma^2 = id$ and $(\sigma\rho)^2 = id$)
But $\sigma(\sigma\rho) = \rho$ which is not in H because $\rho^2 \neq id$.