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3. True or False (If true, then prove it. If false, then give a counterexample.) If H and K are subgroups of the group G, then the intersection of H and K is a subgroup of G.

True closure Take $x, y \in H \cap K$. then $x, y \in H$ which is a group so $xy \in H$ and $x, y \in K$ which is a group so $xy \in K$

Thus $xy \in H \cap K$

id $e \in H \cap K$

assoc G is associative so the operation associates on every subset of G

inverses If $x \in H \cap K$, then H is a group so $x^{-1} \in H$. Also $x \in K$ and K is a group so $x^{-1} \in K$ Thus $x^{-1} \in H \cap K$.

4. True or False (If true, then prove it. If false, then give a counterexample.) If H and K are subgroups of the group G, then the union of H and K is a subgroup of G.

False Here is an example. Let $G = D_4$, $H = \{\sigma, id\}$, $K = \{\sigma\rho, id\}$.

σ and $\sigma\rho \in H \cup K = \{\sigma, \sigma\rho, id\}$ but $\sigma \cdot \sigma\rho = \rho \notin H \cup K$

so $H \cup K$ is not closed, and thus is not a subgroup