Please PRINT your name ______________________________________

No calculators, cell phones, computers, notes, etc.

Make your work correct, complete and coherent.

Please take a picture of your quiz (for your records) just before you turn the quiz in. I will e-mail your grade and my comments to you. I will keep your quiz.

The quiz is worth 5 points. The solutions will be posted on my website later today.

Quiz 7, April 17, 2023

Find the number of cycles of each possible length in S_5. Find all possible orders of elements in S_5. (Try to do this problem without listing all of the elements of S_5.)

Answer: The group S_5 has $5! = 120$ elements. Each element of S_5 is a product of disjoint cycles. In the following discussion a,b,c,d,e are the distinct numbers $1,2,3,4,5$. Each element of S_5 has the form id or (a,b) with $a<b$, or (a,b,c) with $a<b$ and $a<c$, or (a,b,c,d) with $a<b$ and $a<c$ and $a<d$, or (a,b,c,d,e) with $a<b$ and $a<c$ and $a<d$ and $a<e$, or $(a,b)(c,d)$ with $a<b$ and $a<c<d$, or $(a,b)(c,d,e)$ with $a<b$ and $a<c$ and $c<d$ and $c<e$. Now we count the number of elements of each shape. At the same time we record the order of each element of a given shape.

cycle structure	number	order
id	1	1
(a,b)	$\binom{5}{2}=10$	2
(a,b,c)	$2\binom{5}{3}=20$	3
(a,b,c,d)	$3!\binom{5}{4}=30$	4
(a,b,c,d,e)	$4!$	5
$(a,b)(c,d)$	$5(3)=15$	2
$(a,b)(c,d,e)$	$2\binom{5}{2}=20$	6
total	120	

There are $\binom{5}{2}$ ways to choose 2 numbers from $\{1,2,3,4,5\}$. Once you pick a two element subset of $\{1,2,3,4,5\}$, this subset corresponds to one two-cycle.

There are $\binom{5}{3}$ ways to choose 3 numbers from $\{1,2,3,4,5\}$. Once you pick a three element subset of $\{1,2,3,4,5\}$, this subset corresponds to two three-cycles.

There are $4! = 24$ 5-cycles in S_5. Put the smallest number (i.e., 1) first. Each order for $2,3,4,5$ gives rise to a new element of S_5.

There $\binom{5}{4} = 5$ ways to pick a four element subset of $\{1,2,3,4,5\}$. Once you pick a four element subset of $\{1,2,3,4,5\}$, this subset corresponds to $3! = 6$ four-cycles. (Put the smallest number first. Each of arrangements of the remaining three elements gives a new element of S_5.)

There are 5 ways to pick a four element subset of $\{1,2,3,4,5\}$. Once you select the subset $\{a,b,c,d\}$, this subset gives rise to 3 permuations of the form $(i,j)(k,\ell)$; namely $(a,b)(c,d)$, $(a,c)(b,d)$, and $(a,d)(b,c)$.

There are $\binom{5}{2} = 10$ ways to separate $\{1,2,3,4,5\}$ into two subsets so that one subset has two elements and the other subset has 3 elements. The subset with two elements gives rise to one two-cycle; the subset with three elements gives rise to two three cycles.