

Please PRINT your name \_\_\_\_\_

**No calculators, cell phones, computers, notes, etc.**

Make your work correct, complete and coherent.

Please take a picture of your quiz (for your records) just before you turn the quiz in. I will e-mail your grade and my comments to you. I will keep your quiz.

The quiz is worth 5 points. The solutions will be posted on my website later today.

**Quiz 7, November 17, 2022**

Let  $G$  be a finite group, and let  $n$  be a divisor of the order of  $G$ . Prove that if  $H$  is the only subgroup of  $G$  of order  $n$ , then  $H$  must be normal in  $G$ .

**Answer:** Let  $g$  be a fixed, but arbitrary, element of  $G$ . Observe that  $gHg^{-1}$  is a subgroup of  $G$  of order  $n$ . The hypothesis ensures that  $H$  is the only subgroup of  $G$  of order  $n$ . It follows that  $gHg^{-1} = H$ . Indeed,  $gHg^{-1} = H$  for all  $g \in G$ ; therefore  $H$  is a normal subgroup of  $G$ .