Please PRINT your name ______________________________________

No calculators, cell phones, computers, notes, etc.

Make your work correct, complete and coherent.

Please take a picture of your quiz (for your records) just before you turn the quiz in. I will e-mail your grade and my comments to you. I will keep your quiz.

The quiz is worth 5 points. The solutions will be posted on my website later today.

Quiz 2, September 1, 2022

Prove that a non-Abelian group must have at least five distinct elements.

Answer:

• Let G be a group.

• Notice that the identity element commutes with every element of G.

• Notice that if a is an element of G, then a commutes with a and a commutes with the inverse of a.

• Notice that if a and b are elements of G which do not commute, then $ab \neq \text{id}$.

(Indeed, if $ab = \text{id}$, then multiply both sides of the equation on the left by the inverse of a to see that b is equal to the inverse of a. However, a and the inverse of a commute but a and b do not commute.)

• Notice that if a and b are elements of G which do not commute, then $ab \neq a$.

(Indeed, if $ab = a$, then multiply both sides of the equation on the left by the inverse of a to see that $b = \text{id}$. However a and id do commute, but a and b do not commute.)

• Notice that if a and b are elements of G which do not commute, then $ab \neq b$.

(Indeed, if $ab = b$, then multiply both sides of the equation on the right by the inverse of b to see that $a = \text{id}$. However b and id do commute, but b and a do not commute.)

• Now we are ready to write the proof. If a and b are elements of the group G with $ab \neq ba$, then a, b, ab, ba, and id are FIVE different elements of G.