Math 546, Exam 3, Fall, 2004

The exam is worth 50 points.

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Take enough space for each problem. Turn in your solutions in the order: problem 1, problem 2, ... ; although, by using enough paper, you can do the problems in any order that suits you.

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

I will leave your exam outside my office TOMORROW by about 6PM, you may pick it up any time between then and the next class.

I will post the solutions on my website at about 4:00 PM today.

1. (6 points) Define "normal subgroup". Use complete sentences.

2. (6 points) Define "cyclic group". Use complete sentences.

3. (6 points) Define "generator". Use complete sentences.

4. (6 points) What is the order of $(1,1)+H$ in the group $\bar{G}=\frac{G}{H}$, where $G=\mathbb{Z}_6\times\mathbb{Z}_4$ and $H=\{(0,0),(3,0),(0,2),(3,2)\}$? Is $\bar{G}$ a cyclic group? (A small amount of explanation is needed.)

5. (6 points) Find 2 **distinct** elements of order 2 in the group $\bar{G}=\frac{\mathbb{Z}_6\times\mathbb{Z}_4}{<(2,2)>}$. Is $\bar{G}$ a cyclic group? (A small amount of explanation is needed.)

6. (7 points)
   (a) Find an element of order 3 in $\frac{\mathbb{R}}{\mathbb{Z}}$. (A small amount of explanation is needed.)
   (b) Find an element of infinite order in $\frac{\mathbb{R}}{\mathbb{Z}}$. (A small amount of explanation is needed.)

7. (7 points)
   (a) Does there exist a **function** $\varphi\colon \frac{\mathbb{Z}}{9\mathbb{Z}}\to\frac{\mathbb{Z}}{3\mathbb{Z}}$ with $\varphi(a+9\mathbb{Z})=a+3\mathbb{Z}$ for all integers a? Explain thoroughly.
   (b) Does there exist a **function** $\varphi\colon \frac{\mathbb{Z}}{3\mathbb{Z}}\to\frac{\mathbb{Z}}{9\mathbb{Z}}$ with $\varphi(a+3\mathbb{Z})=a+9\mathbb{Z}$ for all integers a? Explain thoroughly.

8. (6 points) Let K be a subgroup of the group G and let N be a normal subgroup of G. Prove that

$$H=\{kn\mid k\in K \text{ and } n\in N\}$$

is a subgroup of G.