Math 546, Exam 1, Fall, 2004

The exam is worth 50 points.

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Take enough space for each problem. Turn in your solutions in the order: problem 1, problem 2, ... ; although, by using enough paper, you can do the problems in any order that suits you.

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

I will leave your exam outside my office tomorrow by about noon, you may pick it up any time between then and the next class.

I will post the solutions on my website at about 4:00 PM today.

1. (6 points) Define "group". Use complete sentences.

2. (6 points) Define "centralizer". Use complete sentences.

3. (6 points) Exhibit a group G and two elements a and b of G with $(ab)^3 \neq a^3 b^3$.

4. (7 points) Let G be a group and let a and b be elements of G with $a^{-1}b^{-1} = (ab)^{-1}$. Prove that $ab = ba$.

5. (7 points) Let G be an abelian group and let H be the subset $H = \{g \in G \mid g^3 = \text{id}\}$ of G . Prove that H is a subgroup of G .

6. (6 points) List the elements that are in the centralizer of $\rho^3 \sigma$ in D_4 . (I do not need to see any explanation.)

7. (6 points) Let $G = (\mathbb{Q} \setminus \{0\}, *)$, where $a * b = |ab|$. Is G a group? Explain.

8. (6 points) Let $G = (\mathbb{Q} \setminus \{0\}, *)$, where $a * b = \frac{a}{b}$. Is G a group? Explain.