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3. Let G be an abelian group with identity element e. Let

$$H = \{x \in G \mid x^2 = e\}.$$

Prove that H is a subgroup of G.

$e \in H$ because $e^2 = e$

closed Take x and $y \in H$. $(xy)^2 = xyxy = x^2y^2 = e$

G is abelian $x, y \in H$

thus $xy \in H$

inverses Take $x \in H$ we know $x^2 = e$ $\therefore$ $xx = e$
multiply by x^{-1} twice $e = x^{-1}x^{-1}$ Thus $(x^{-1})^2 = e$
and $x^{-1} \in H$.

4. Let G be a group with identity element e. Suppose that a, b, and c are elements of G with $c * b * a = e$. Prove that $b * a * c$ is also equal to e.

we know $cba = e$.

multiply by c^{-1} $ba = c^{-1}$

multiply by c $bac = e$.