

PRINT Your Name: _____

There are 7 problems on 4 pages. The exam is worth a total of 50 points. Problem 1 is worth 8 points. The other problems are worth 7 points each.

1. TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.) If H and K are subgroups of a group G , then the intersection $H \cap K$ is also a subgroup of G .

True. $H \cap K$ is nonempty because $e \in H$ and $e \in K$
closed Take x and $y \in H \cap K$. Then $x, y \in H$ and H is closed because it is a group so $xy \in H$, similarly $x, y \in K$ and K is closed so $xy \in K$ so $xy \in H \cap K$
inverse Take $x \in H \cap K$. Then $x \in H$ and H is a group so $x^{-1} \in H$. In a similar manner $x \in K$ and K is a group so $x^{-1} \in K$. Thus, $x^{-1} \in H \cap K$.

2. TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.) If H and K are subgroups of a group G , then the union $H \cup K$ is also a subgroup of G .

False $G = \mathbb{Z}$ $H = 2\mathbb{Z}$ $K = 3\mathbb{Z}$ $H \cup K$ is not closed;
 for example $2 \in H \cup K$ $3 \in H \cup K$ but $2+3=5 \notin H \cup K$.