Math 546, Exam 1, Spring, 2023

You should KEEP this piece of paper. Write everything on the **blank paper provided**. Return the problems **in order** (use as much paper as necessary), use **only one side** of each piece of paper. Number your pages and write your name on each page. Take a picture of your exam (for your records) just before you turn the exam in. I will e-mail your grade and my comments to you. I will keep your exam. **Fold your exam in half** before you turn it in.

No calculators, cell phones, computers, notes, etc.

Make your work correct, complete, and coherent.

The exam is worth 50 points. Each problem is worth 10 points.

The solutions will be posted later today.

(1) Let H be the subgroup of $(\mathbb{Z},+)$ generated by 2. (Recall that $(\mathbb{Z},+)$ is the group of integers under addition.) What are the elements of H? Explain.

(2) Let H be the subgroup of $(\mathrm{GL}_2(\mathbb{R}),\times)$ generated by $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. (Recall that $(\mathrm{GL}_2(\mathbb{R}),\times)$ is the group of 2×2 invertible matrices with real entries under multiplication.) What are the elements of H? Explain.

(3) Let $(\mathbb{Z},*)$ be the set of integers with operation $a * b = \max\{a,b\}$. (In other words, $a * b$ is equal to the maximum of a and b.) Is $(\mathbb{Z},*)$ a group? Explain.

(4) Recall that $\mathcal{G}$ is the group of rigid motions of the xy-plane with operation composition. Let ρ be the element of $\mathcal{G}$ which fixes the orign and rotates the xy-plane counterclockwise by 72 degrees. Let H be the subgroup of $\mathcal{G}$ which is generated by ρ. Write the multiplication table for H.

(5) (a) Is it possible for a group to be cyclic, but not Abelian? Explain.

(b) Is it possible for a group to be Abelian, but not cyclic? Explain.