Math 546, Exam 3, Fall, 1994

PRINT Your Name:________________________________

There are 7 problems on 3 pages. The exam is worth a total of 50 points. Problem 3 is worth 8 points. The other problems are worth 7 points each.

1. DEFINE *normal subgroup.*

2. DEFINE *kernel.*

3. STATE LaGrange's theorem.

4. TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.) Let G be a group and let a be a fixed element of G. If $\rho_a\colon G \to G$, is the function which is given by $\rho_a(g) = ga$ for all $g \in G$, then ρ_a is a permutation of the set G.

5. TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.) Let G be a group and let a be a fixed element of G. If $\gamma_a\colon G \to G$, is the function which is given by $\gamma_a(g) = a^{-1}ga$ for all $g \in G$, then γ_a is a homomorphism.

6. TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.) If $(G, *)$ is a group, then the function $\phi\colon G \to G$, which is given by $\phi(g) = g*g$ is a group homomorphism.

7. TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.) Let a, b, and c be elements of a group G and let H be a subgroup of G. If $aH = bH$, then $acH = bcH$.