Math 546, Exam 1, Fall , 1994

PRINT Your Name:__________________________________

There are 7 problems on 4 pages. The exam is worth a total of 50 points. Problem 1 is worth 8 points. The other problems are worth 7 points each.

1. TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.) If H and K are subgroups of a group G, then the intersection $H \cap K$ is also a subgroup of G.

2. TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.) If H and K are subgroups of a group G, then the union $H \cup K$ is also a subgroup of G.

3. Let G be an abelian group with identity element e. Let

$$H = \{x \in G \mid x^2 = e\}.$$

Prove that H is a subgroup of G.

4. Let G be a group with identity element e. Suppose that a, b, and c are elements of G with $c * b * a = e$. Prove that $b * a * c$ is also equal to e.

5. Let $\mathbb{R}^*$ represent the set of nonzero real numbers. Define a binary operation $*$ on $\mathbb{R}^*$ by $a * b = b/a$. Is $(\mathbb{R}^*, *)$ a group? If so prove it. If not, show why not.

6. Let G be a group. Let

$$H = \{x \in G \mid \ xy = yx \text{ for all } y \in G\}.$$

Prove that H is a subgroup of G.

7. Let G be a group with identity element e. Suppose that $x^2 = e$ for all $x \in G$. Prove that G is an abelian group.