

Solution to the Quiz for June 17, 2003

Let U and V be subspaces of \mathbb{R}^n . Prove that the intersection, $U \cap V$, is also a subspace of \mathbb{R}^n .

Zero is in $U \cap V$: We know that zero is in U because U is a vector space. We know that zero is in V because V is a vector space. Thus, zero is in $U \cap V$.

$U \cap V$ is closed under addition: Take x and y from $U \cap V$. We know that $x, y \in U$ and U is a vector space. It follows that $x + y \in U$. We know that $x, y \in V$ and V is a vector space. It follows that $x + y \in V$. Now we know that $x + y \in U \cap V$.

$U \cap V$ is closed under scalar multiplication: Take x in $U \cap V$ and $r \in \mathbb{R}$. We know that $x \in U$, r is a scalar, and U is a vector space. It follows that $rx \in U$. We know that $x \in V$, r is a scalar, and V is a vector space. It follows that $rx \in V$. Now we know that $rx \in U \cap V$.