Solution to the Quiz for June 10, 2003

Suppose that the vectors $\{v_1, v_2, v_3\}$ in $\mathbb{R}^m$ are linearly independent. Prove that the vectors $\{v_1, v_1 + v_2, v_1 + v_2 + v_3\}$ are also linearly independent.

Suppose c_1, c_2, c_3 are numbers with

$$c_1 v_1 + c_2(v_1 + v_2) + c_3(v_1 + v_2 + v_3) = 0.$$

It follows that

$$(c_1 + c_2 + c_3)v_1 + (c_2 + c_3)v_2 + c_3 v_3 = 0.$$

The vectors v_1, v_2, v_3 are linearly independent. It follows that

$$\begin{aligned} c_1 + c_2 + c_3 &= 0, \\ c_2 + c_3 &= 0, \\ c_3 &= 0. \end{aligned}$$

Read from the bottom up to see that $c_3 = 0$, $c_2 = 0$, and $c_1 = 0$. We conclude that v_1, $v_1 + v_2$, $v_1 + v_2 + v_3$ are linearly independent.