Solution to the Quiz for March 19, 2003

Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the function that maps each point in $\mathbb{R}^2$ to its reflection with respect to the x-axis. Give a formula for T and show that T is a linear transformation.

Observe that $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ -y \end{bmatrix}$. In other words, $T(v) = Mv$ (be sure to read the last equation "T of v equals M times v"), where $M = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. We saw in class that matrix multiplication always is a linear transformation. (Of course, you may verify this last assertion again. If M is a matrix, v and w are column vectors, and r is a number, then $M(v+w) = Mv + Mw$ holds because this is distribution and $M(rv) = rMv$, because this is the way that scalar multiplication interacts with matrix multiplication.)