Solution to the Quiz for March 19, 2003

Express $v = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ as a linear combination of

$$u_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad u_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \quad \text{and} \quad u_3 = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}.$$

(The problem is especially easy if you take advantage of the fact that u_1, u_2, u_3 are an orthogonal set of vectors.) Check your answer!

We solve $v = c_1 u_1 + c_2 u_2 + c_3 u_3$. Multiply both sides of the equation (on the left) by u_1^{T} to see that $2 = 3c_1$. Mutilpy by u_2^{T} to see that $-1 = 2c_2$. Multiply by u_3^{T} to see that $1 = 6c_3$. We conclude that

$$\boxed{v = \frac{2}{3}u_1 - \frac{1}{2}u_2 + \frac{1}{6}c_3.}$$

This is correct because:

$$\frac{2}{3}u_1 - \frac{1}{2}u_2 + \frac{1}{4}c_3 = \frac{2}{3}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \frac{1}{2} + \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + \frac{1}{6}\begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = v.\checkmark$$