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9. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If A and B are 2×2 nonsingular matrices, then $A + B$ is a nonsingular matrix.

False $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ are nonsingular (because their columns are linearly independent, But

$A+B = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$ is singular

because $0\begin{bmatrix} 2 \\ 0 \end{bmatrix} + 1\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

10. Define "linearly independent".

The vectors $v_1, \ldots, v_p$ in $\mathbb{R}^m$ are linearly independent if the only solution of $x_1 v_1 + \cdots + x_p v_p = 0$ is $x_1 = x_2 = \cdots = x_p = 0$.