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Quiz for March 1, 2011

Let

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 2 & 5 & 3 & -1 \\ 2 & 2 & 0 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix}.$$

Find a basis for the null space of A.

ANSWER: To find a basis for the null space of A, we solve $Ax = 0$. In other words, we apply Elementary Row Operations to A. Apply $R2 \mapsto R2 - 2R1$ and $R3 \mapsto R3 - 2R1$ to get:

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix}.$$

Apply $R1 \mapsto R1 - 2R2$, $R3 \mapsto R3 + 2R2$, and $R4 \mapsto R4 - R2$ to get

$$\begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The solution set of $Ax = 0$ is the set of all vectors x with

$$\begin{array}{rcrr} x_1 & = & x_3 & -2x_4 \\ x_2 & = & -x_3 & +x_4 \\ x_3 & = & x_3 & \\ x_4 & = & & x_4 \end{array}$$

In other words the null space of A is the set of linear combinations of

$$\begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix}.$$

These two vectors are linearly independent (look at rows 3 and 4); so our answer is

$$\boxed{v_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix}.}$$