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Quiz for September 5, 2006

Suppose A and B are 2×2 matrices with A not equal to the zero matrix and $A^2 = AB$. Does A have to equal B? If yes, then prove it. If no, then give an example.

ANSWER: $\boxed{\text{NO}}$. Here is one example. Take $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 75 & 144 \\ 0 & 0 \end{bmatrix}$. We see that

$$A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

and

$$AB = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 75 & 144 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

So, $A^2 = AB$, A is not the zero matrix, and $A \neq B$.