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Quiz for September 26, 2006

Let U and V be subspaces of $\mathbb{R}^n$. Prove that the intersection $U \cap V$ is a subspace of $\mathbb{R}^n$.

ANSWER:

Zero vector: The zero vector is in U since U is a subspace of $\mathbb{R}^n$. The zero vector in V since V is a subspace of $\mathbb{R}^n$. Therefore, the zero vector is in the intersection $U \cap V$.

Closed under addition: Consider vectors x and y in the intersection $U \cap V$. The vectors x and y are both in the subspace U. The subspace U is closed under addition. It follows that the sum $x + y$ is in U. The vectors x and y are both in the subspace V. The subspace V is closed under addition. It follows that the sum $x + y$ is in V. Combine these two conclusions to see that the sum $x + y$ is in the intersection $U \cap V$.

Closed under scalar multiplication: Consider a vector x in $U \cap V$ and a scalar $c \in \mathbb{R}$. The vector x is in the subspace U and U is closed under scalar multiplication; thus, cx is in U. The vector x is in the subspace V and V is closed under scalar multiplication; thus, cx is in V. Combine these two conclusions to see that the cx is in the intersection $U \cap V$.