PRINT Your Name:_____

Quiz for September 19, 2006

(a) Give an example of non-singular matrices A and B with $A+B$ singular.

(b) Give an example of singular matrices A and B with $A+B$ non-singular.

ANSWER:

(a) The matrices $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ are non-singular (because the columns are linearly independent in each matrix), but the sum $A+B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ is singular (since the sum sends the non-zero vector $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ to zero.)

(b) The matrices $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ are singular (since $A \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $B \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ are both zero and neither $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ nor $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is zero); but the sum $A+B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is non-singular (because the columns are linearly independent).