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Quiz for June 5, 2006

Let v_1 , v_2 , and v_3 be linearly independent vectors in $\mathbb{R}^m$. Prove that the vectors $v_1, v_1+v_2, v_1+v_2+v_3$ are also linearly independent.

ANSWER: Suppose c_1, c_2, c_3 are numbers with

$$c_1(v_1) + c_2(v_1+v_2) + c_3(v_1+v_2+v_3) = 0. \tag{*}$$

We must show that c_1 , c_2 , and c_3 must be zero. Rewrite (*) as

$$(c_1+c_2+c_3)v_1 + (c_2+c_3)v_2 + c_3v_3 = 0.$$

The vectors v_1 , v_2 , and v_3 are linearly independent; so,

$$\begin{cases} c_1+c_2+c_3 = 0 \\ c_2+c_3 = 0 \\ c_3 = 0. \end{cases}$$

Read from the bottom equation up to learn to c_3 must be zero; hence, c_2 must be zero; and finally, c_1 must be zero.