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Quiz for May 31, 2005

Find all values a for which the system

$$\begin{aligned} x_1 + 2x_2 &= -3 \\ ax_1 - 2x_2 &= 5 \end{aligned}$$

has no solution. Explain thoroughly.

ANSWER: We apply the technique of Guassain Elimination to

$$\left[\begin{array}{cc|c} 1 & 2 & -3 \\ a & -2 & 5 \end{array}\right]$$

Replace $R_2 \mapsto R_2 - aR_1$ to get:

$$\left[\begin{array}{cc|c} 1 & 2 & -3 \\ 0 & -2-2a & 5+3a \end{array}\right]$$

If $-2-2a \neq 0$, then the system of equations has exactly one solution. If $-2-2a = 0$ and $5+3a = 0$ (this never happens), then the system of equations has infinitely many solutions. If $-2-2a = 0$ and $5+3a \neq 0$, then the system of equations has no solution. We conclude that:

$$\boxed{\begin{array}{c} \text{If } a = -1\text{, then the system has no solutuion.} \\ \text{Otherwise, the system of equations has a unique solution.} \end{array}}$$