Quiz for February 28, 2003

Let A be the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 3 & 5 & 8 & -2 \\ 1 & 1 & 2 & 0 \end{bmatrix}.$$

(a) Find a basis for the null space of A.
(b) Find a basis for the column space of A.
(c) Find a basis for the row space of A.
Replace row 2 with row 2 minus 3 times row 1.
Replace row 3 with row 3 minus row 1.

$$\begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & -1 & -1 & 1 \\ 0 & -1 & -1 & 1 \end{bmatrix}.$$

Replace row 1 with row 1 plus 2 row 2.
Replace row 3 with row 3 minus row 2.

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & -1 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Replace row 2 by minus row 2.

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The null space of A is the set of all $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ with

$$\begin{aligned} x_1 &= -x_3 - x_4 \\ x_2 &= -x_3 + x_4 \\ x_3 &= x_3 \\ x_4 &= x_4 \end{aligned}.$$

So, a basis for the null space of A is

(a) $\begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$

A basis for the column space of A is

(b) $\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}$

A basis for the row space of A is

(c) $\quad \begin{bmatrix} 1 & 0 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 1 & -1 \end{bmatrix}$