PRINT Your Name:____________________________________

Quiz for February 22, 2011

Let A and B be $n \times n$ matrices. Let V_1 be the null space of A, V_2 be the null space of B, and V_3 be the null space of $A+B$. Prove that $V_1 \cap V_2$ is contained in V_3. Recall that the intersection, $V_1 \cap V_2$, of the two sets V_1 and V_2 is the set which contains those elements that are in **both** V_1 and V_2.

ANSWER: Take $v \in V_1 \cap V_2$. We have chosen v in the null space of A and in the null space of B. We will now show that v is in the null space of $A+B$. We compute

$$(A+B)v = Av + Bv = 0 + 0 = 0.$$

The first equality is distribution. The second equality follows since v is in the null space of A and v is in the null space of B. We have shown that v is in the null space of $A+B$; thus, $v \in V_3$.