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Quiz for November 15, 2005

Let $A = \begin{bmatrix} 1 & 0 \\ 10 & 2 \end{bmatrix}$. Find a diagonal matrix D and an invertible matrix S with $A = SDS^{-1}$.

Answer: The eigenvalues of A are $\lambda = 1, 2$. The vector $v_1 = \begin{bmatrix} 1 \\ -10 \end{bmatrix}$ is an eigenvector of A belonging to $\lambda = 1$ because $Av_1 = \begin{bmatrix} 1 & 0 \\ 10 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -10 \end{bmatrix} = \begin{bmatrix} 1 \\ -10 \end{bmatrix} = v_1$. The vector $v_2 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ is an eigenvector of A belonging to $\lambda = 2$ because $Av_2 = \begin{bmatrix} 1 & 0 \\ 10 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} = 2v_2$. Let

$$\boxed{S = \begin{bmatrix} 1 & 0 \\ -10 & 2 \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}.}$$

We have just calculated that $AS = SD$. In other words, $A = SDS^{-1}$.