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Quiz for October 11, 2005

Let A be a 5×4 matrix. What is the largest possible value for the rank of A? What is the smallest possible value for the nullity of A? Explain.

ANSWER: The rank of A is the dimension of the columns space of A. The column space of A is spanned by the four columns of A. Some subset of these four columns forms a basis for the column space of A; so the dimension of the column space of A is less than or equal to 4. We notice that this maximum value is attained sometimes, for example in

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The rank of A plus the nullity of A is equal to the number of columns of A. The nullity is minimized when the rank is maximized. So the minimum possible nullity for A is 0. Indeed, the indicated matrix has nullity zero.