

Let v_1, v_2, v_3 be linearly dependent vectors in \mathbb{R}^m . Prove that the vectors v_1, v_2, v_3, v_4 are linearly dependent for all vectors v_4 in \mathbb{R}^m .

ANSWER: Fix an arbitrary vector v_4 in \mathbb{R}^m . The first sentence guarantees that there are numbers a_1, a_2, a_3 , at least one of which is non-zero, with $a_1v_1 + a_2v_2 + a_3v_3 = 0$. Thus, we have numbers $a_1, a_2, a_3, 0$, at least one of which is not zero, and $a_1v_1 + a_2v_2 + a_3v_3 + 0v_4 = 0$. We conclude that the vectors v_1, v_2, v_3, v_4 are linearly dependent.