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11. (4 points) Define "eigenvalue". Use complete sentences.

The number c is an eigenvalue for the square matrix A if there exists a vector $v \neq 0$ with $Av = cv$.

12. (4 points) Diagonalize the matrix $A = \begin{bmatrix} 5 & -6 \\ 3 & -4 \end{bmatrix}$.

$\det(A-\lambda I) = \begin{vmatrix} 5-\lambda & -6 \\ 3 & -4-\lambda \end{vmatrix} = (5-\lambda)(-4-\lambda)+18 = \lambda^2-\lambda-2 = (\lambda-2)(\lambda+1) \quad \lambda = 2,-1$

$\underline{\lambda=2}$ $A-2I = \begin{bmatrix} 3 & -6 \\ 3 & -6 \end{bmatrix}$ $\begin{matrix} R2 \mapsto R2-R1 \\ R1 \mapsto \frac{1}{3}R1 \end{matrix}$ $\begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}$ $\begin{matrix} x_1 = 2x_2 \\ x_2 = x_2 \end{matrix}$ evector $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $A\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} = 2\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ ✓

$\underline{\lambda = -1}$ $A+I = \begin{bmatrix} 6 & -6 \\ 3 & -3 \end{bmatrix}$ $\begin{matrix} R2 \mapsto R2-\frac{1}{2}R1 \\ R1 \mapsto \frac{1}{6}R1 \end{matrix}$ $\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$ $\begin{matrix} x_1 = x_2 \\ x_2 = x_2 \end{matrix}$ evector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $A\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5-6 \\ 3-4 \end{bmatrix} = \begin{pmatrix} -1 \\ -1 \end{pmatrix} = -1\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Let $P = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$

We just calculated $AP = PD$

So $A = PDP^{-1}$