

4. (4 points) Define "linearly independent". Use complete sentences.

The vectors  $v_1, \dots, v_n$  are linearly independent if the only numbers  $c_1, \dots, c_n$  with  $c_1v_1 + \dots + c_nv_n = 0$  are  $c_1 = c_2 = \dots = c_n = 0$ .

5. (4 points) Define "linear transformation". Use complete sentences.

The function  $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$  is a linear transformation if  $T(v+w) = T(v) + T(w)$  and  $T(cv) = cT(v)$  for all  $v, w \in \mathbb{R}^n$  and all  $c \in \mathbb{R}$ .

6. (4 points) Define "one-to-one". Use complete sentences.

The function  $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$  is one-to-one if for each  $b \in \mathbb{R}^m$  there is at most one  $v \in \mathbb{R}^n$  with  $T(v) = b$ .

7. (4 points) Define "dimension". Use complete sentences.

If the vectors  $v_1, \dots, v_n$  are a basis for the vector space  $V$ , then  $n$  is the dimension of  $V$ .

8. (4 points) Define "column space". Use complete sentences.

The column space of a matrix is the vector space which is spanned by the columns of the matrix.