Final 544 Summer 2001

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PRINT Your Name:________________________________

There are 14 problems on 6 pages. The exam is worth 100 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible. **NO CALCULATORS.** Your grade for the course will be available on VIP by Wednesday July 11.

1. (16 points) Let A be an $n \times n$ matrix. List 8 statements that are equivalent to the statement "A is invertible".

(1) The rows of A are linearly independent.
(2) The columns of A are linearly independent.
(3) The columns of A span $\mathbb{R}^n$
(4) The columns of A are a basis for $\mathbb{R}^n$
(5) The dimension of the column space of A is n.
(6) The only solution of $Ax=0$ is $x=0$.
(7) $Ax=b$ has exactly one solution for each b in $\mathbb{R}^n$
(8) $Ax=b$ has at least one solution for each b in $\mathbb{R}^n$
(9) $Ax=b$ has at most one solution for each b in $\mathbb{R}^n$.
(10) The null space of A consists of the zero vector.
(11) There exists a matrix B with $AB=I_n$
(12) There exists a matrix B with $BA=I_n$
(13) A^T is invertible
(14) $\lambda=0$ is not an eigenvalue of A.
(15) The linear transformation $x \mapsto Ax$ is one-to-one.
(16) The linear transformation $x \mapsto Ax$ is onto.

2. (4 points) Define "span". Use complete sentences.

The span of the vectors $v_1,\dots,v_n$ is the set of all linear combinations of $v_1,\dots,v_n$.

Or

The vectors $v_1,\dots,v_n$ span the vector space V if $v_1,\dots,v_n$ are in V and every vector in V is a linear combination of $v_1,\dots,v_n$.

3. (4 points) Define "linear combination". Use complete sentences.

The vector v is a linear combination of the vectors $v_1,\dots,v_n$ if $v=c_1v_1+\dots+c_nv_n$ for some numbers $c_1,\dots,c_n$.