17. (5 points) True or False. (If true, give a proof. If false, give a counter example.) If v_1, v_2, v_3 are linearly independent vectors in $\mathbb{R}^4$ and $T\colon \mathbb{R}^4 \to \mathbb{R}^4$ is a linear transformation, then $T(v_1), T(v_2), T(v_3)$ are linearly independent vectors in $\mathbb{R}^4$.

FAlse Let T be the linear transformation which sends every vector to 0.

I can have $v_1 = \begin{bmatrix}1\\0\\0\\0\end{bmatrix}$ $v_2 = \begin{bmatrix}0\\1\\0\\0\end{bmatrix}$ $v_3 = \begin{bmatrix}0\\0\\1\\0\end{bmatrix}$. These are l.i

but $T(v_1) = T(v_2) = T(v_3) = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$ These are l.d.

18. (5 points) True or False. (If true, give a proof. If false, give a counter example.) If v_1, v_2, v_3 are linearly dependent vectors in $\mathbb{R}^4$ and $T\colon \mathbb{R}^4 \to \mathbb{R}^4$ is a linear transformation, then $T(v_1), T(v_2), T(v_3)$ are linearly dependent vectors in $\mathbb{R}^4$.

True If there are 3 numbers c_1, c_2, c_3 not all zero with $c_1v_1 + c_2v_2 + c_3v_3 = 0$, then

$T(c_1v_1 + c_2v_2 + c_3v_3) = T(0)$

So $c_1T(v_1) + c_2T(v_2) + c_3T(v_3) = 0$

So I have a non-trivial linear combination of $T(v_1), T(v_2), T(v_3)$ which equals 0

$T(v_1), T(v_2), T(v_3)$ are l.d.