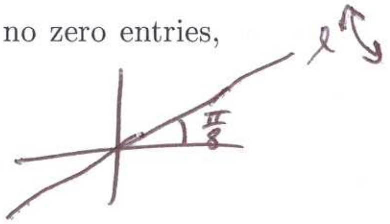
12. (5 points) Give an example of a two by two matrix A, with no zero entries, such that A^2 is equal to the identity matrix.



Any reflection matrix will do. I reflect across

$$A = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix}$$

Notice $A^2 = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

13. (5 points) Let

$$A = \begin{bmatrix} 1 & 1 & 0 & -1 \\ 1 & -1 & 0 & -1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & -1 & 1 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}.$$

Solve $Ax = b$. (You may do the problem any way you wish; however, you may find it helpful to notice that the columns of A form an orthogonal set.) **Check your answer.**

$Ax = b$ I solve $A^TAx = A^Tb$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ -1 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & -1 \\ 1 & -1 & 0 & -1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & -1 & 1 \end{bmatrix} x = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ -1 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} x = \begin{bmatrix} 10 \\ -1 \\ -1 \\ 4 \end{bmatrix} \quad \text{So} \quad x = \begin{bmatrix} \frac{5}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \\ 1 \end{bmatrix}$$

<u>Check</u> $Ax = \frac{1}{2} \begin{bmatrix} 1 & 1 & 0 & -1 \\ 1 & -1 & 0 & -1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ -1 \\ -1 \\ 2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 2 \\ 4 \\ 6 \\ 8 \end{bmatrix}$ ✓