6

10. (5 points) Let

$$A = \begin{bmatrix} 1 & 2 \\ 1 & -1 \\ 1 & -1 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}.$$

Find the least squares solution for $Ax = b$. **Be sure to check your answer,** call it $\hat{x}$, by verifying that $b - A\hat{x}$ is orthogonal to the column space of A.

We solve $A^TA\hat{x} = A^Tb$

$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}\hat{x} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & -1 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$ $\begin{bmatrix} 3 & 0 \\ 0 & 6 \end{bmatrix}\hat{x} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$

So $\hat{x} = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{6} \end{bmatrix}\begin{bmatrix} 3 \\ 6 \end{bmatrix} = \boxed{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}$

Check

$b - A\hat{x} = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix} - \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

I am ⊥ to the columns of A!

11. (5 points) Is it possible that all solutions of a homogeneous system of ten linear equations in twelve variables are multiples of one fixed non-zero solution? Explain.

The question asks that $Ax = 0$ when A is 10×12

rank + nullity = the # of columns and rank ≤ 10

so dim null space A ≥ 2 so the answer is NO!

It is not possible for all solutions of $Ax = 0$ to be multiples of one fixed solution.