9. (5 points) Find an orthogonal set which is a basis for the null space of $A = [1 \quad -1 \quad 1 \quad -1]$.

$v_1 = \begin{bmatrix} +1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ $v_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ $v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ is a basis for the nullspace of A

Let $u_1 = v_1$

Let $u_2' = v_2 - \frac{v_2 u_1}{u_1 u_1} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} - \frac{-1}{2} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \\ +\frac{1}{2} \\ 1 \\ 0 \end{bmatrix}$

Let $u_2 = 2u_2' = \begin{bmatrix} -1 \\ +1 \\ 2 \\ 0 \end{bmatrix}$ Notice $u_2 \in$ nullspace A

$u_2 \perp u_1$

Let $u_3' = v_3 - \frac{v_3 u_1}{u_1 u_1} u_1 - \frac{v_3 u_2}{u_2 u_2} u_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} - \frac{-1}{6} \begin{bmatrix} -1 \\ 1 \\ 2 \\ 0 \end{bmatrix}$

$= \frac{1}{6} \begin{bmatrix} 6 & -3 & -1 \\ -3 & +1 & \\ & +2 & \\ 6 & & \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 2 \\ -2 \\ +2 \\ 6 \end{bmatrix}$

Let $u_3 = \frac{6}{2} u_3' = \begin{bmatrix} 1 \\ -1 \\ +1 \\ 3 \end{bmatrix}$

check $u_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$, $u_2 = \begin{bmatrix} -1 \\ 1 \\ 2 \\ 0 \end{bmatrix}$, $u_3 = \begin{bmatrix} 1 \\ -1 \\ +1 \\ 3 \end{bmatrix}$

are an orthog set in nullspace A