8. (9 points) Let

$$A = \begin{bmatrix} 1 & 0 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \\ 1 & 0 & 2 & 4 & 6 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}.$$

Solve $Ax = b$. Find a basis for the null space of A. Find a basis for the column space of A. Find a basis for the row space of A.

$\left[\begin{array}{ccccc|c} 1 & 0 & 3 & 6 & 9 & 4 \\ 0 & 0 & 1 & 2 & 3 & 1 \\ 1 & 0 & 2 & 4 & 6 & 3 \end{array}\right]$ $R3 \to R3-R1$ $\left[\begin{array}{ccccc|c} 1 & 0 & 3 & 6 & 9 & 4 \\ 0 & 0 & 1 & 2 & 3 & 1 \\ 0 & 0 & -1 & -2 & -3 & -1 \end{array}\right]$ $R3 \to R3+R2$, $R1 \to R1-3R2$ $\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 2 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$

$x_1 = 1$
$x_2 = x_2$
$x_3 = 1 - 2x_4 - 3x_5$
$x_4 = x_4$
$x_5 = x_5$

← the general solution of $Ax = b$

Notice $A \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}$

$\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -3 \\ 0 \\ 1 \end{bmatrix} \right\}$

↑ basis for null space A

$\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} \right\}$

↑ basis for col space A

$\{ [1\,0\,0\,0\,0], [0\,0\,1\,2\,3] \}$

↑ basis for row space A