7. (8 points) Let $A = \begin{bmatrix} 7 & 6 \\ -3 & -2 \end{bmatrix}$. Diagonalize A. Find a matrix B with $B^2 = A$. **Check your answer.**

$0 = |A - \lambda I| = \det\begin{pmatrix} 7-\lambda & 6 \\ -3 & -2-\lambda \end{pmatrix} = (7-\lambda)(-2-\lambda) + 18 = \lambda^2 - 5\lambda + 4 = (\lambda - 1)(\lambda - 4)$

so $\lambda = 1$ & $\lambda = 4$ are the e values

e-space for $\lambda = 1$ $\quad A - I = \begin{bmatrix} 6 & 6 \\ -3 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ $\quad x_1 = -x_2$, $x_2 = x_2$ $\quad$ evector $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

e-space for $\lambda = 4$ $\quad A - 4I \begin{bmatrix} 3 & 6 \\ -3 & -6 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$ $\quad x_1 = -2x_2$, $x_2 = x_2$ $\quad$ evector $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

check $\quad A \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$

so $A = PDP^{-1}$ for $P = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$ $\qquad P^{-1} = \frac{1}{+1}\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} = \begin{matrix} +1 & +2 \\ -1 & -1 \end{matrix}$

Let $B = P \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} P^{-1} = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} -1 & -4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$

$= \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$

$\begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ -3 & -2 \end{bmatrix} \leftarrow$ check