~~66~~ 85

10. (10 points) Find the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}.$$

$\det(A - \lambda I) = \det\begin{pmatrix} 1-\lambda & 2 \\ 0 & 3-\lambda \end{pmatrix} = (1-\lambda)(3-\lambda)$

evalues are 1, 3

$\lambda = 1$ $\quad A - I = \begin{bmatrix} 0 & 2 \\ 0 & 2 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ evector $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$A\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ ✓

$\lambda = 3$ $\quad A - 3I = \begin{bmatrix} -2 & 2 \\ 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$ evector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$A\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$ ✓

The evectors that belong to $\lambda = 1$ are all multiples of $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

The evectors that belong to $\lambda = 3$ are all multiples of $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.