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6. (10 points) Solve the system of equations which corresponds to the following augmented matrix:

$$\left[\begin{array}{ccccc|c} 1 & 2 & 0 & 4 & 0 & 1 \\ 0 & 0 & 1 & 3 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{array}\right].$$

No solution

7. (20 points) Let

$$A = \begin{bmatrix} 1 & 0 & 2 & 3 & 4 & 0 & 5 & 0 \\ 1 & 0 & 2 & 3 & 4 & 0 & 11 & 0 \\ 1 & 0 & 2 & 3 & 4 & 0 & 11 & 1 \end{bmatrix}.$$

(a) Find a basis for the row space of A.
(b) Find a basis for the column space of A.
(c) Find a basis for the null space of A.
(d) What is the dimension of the null space of A?
(e) What is the dimension of the column space of A?

$R2 \mapsto R2 - R1$, $R3 \mapsto R3 - R1$ $\begin{bmatrix} 1 & 0 & 2 & 3 & 4 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 6 & 1 \end{bmatrix}$

$R3 \mapsto R3 - R2$ $\begin{bmatrix} 1 & 0 & 2 & 3 & 4 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

$R2 \mapsto \frac{1}{6} R2$ $\begin{bmatrix} 1 & 0 & 2 & 3 & 4 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

$R1 \mapsto R1 - 5R2$ $\begin{bmatrix} 1 & 0 & 2 & 3 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

(a) $[1\ 0\ 2\ 3\ 4\ 0\ 0\ 0]$, $[0\ 0\ 0\ 0\ 0\ 0\ 1\ 0]$, $[0\ 0\ 0\ 0\ 0\ 0\ 0\ 1]$

(b) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 11 \\ 11 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

(c) the null space is

$x_1 = -2x_3 - 3x_4 - 4x_5$
$x_2 = x_2$
$x_3 = x_3$
$x_4 = x_4$
$x_5 = x_5$
$x_6 = x_6$
$x_7 = 0$
$x_8 = 0$

$\begin{bmatrix} -4 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

(d) 5

(e) 3