~~60~~ 79

16. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If A and B are 2×2 nonsingular matrices, then $A + B$ is a nonsingular matrix.

False $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ are non-singular

but $A + B = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$ which is singular

17. Find an orthogonal set which is a basis for the null space of $[1 \quad 2 \quad 1 \quad 2]$.

One basis for the nullspace is $w_1 = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ $\underset{w_2}{\begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}}$ $\begin{bmatrix} -2 \\ 0 \\ 0 \\ 1 \end{bmatrix} = w_3$

Let $u_1 = w_1$

Let $u_2' = w_2 - \frac{u_1^T w_2}{u_1^T u_1} u_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} - \frac{2}{5}\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{1}{5} \\ -\frac{2}{5} \\ \frac{5}{5} \\ 0 \end{bmatrix}$, Let $u_2 = \begin{bmatrix} -1 \\ -2 \\ 5 \\ 0 \end{bmatrix}$

Let $u_3' = w_3 - \frac{u_1^T w_3}{u_1^T u_1} u_1 - \frac{u_2^T w_3}{u_2^T u_2} u_2 = \begin{bmatrix} -2 \\ 0 \\ 0 \\ 1 \end{bmatrix} - \frac{4}{5}\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} - \frac{2}{30}\begin{bmatrix} -1 \\ -2 \\ 5 \\ 0 \end{bmatrix} = \frac{5}{15}\begin{bmatrix} -1 \\ -2 \\ -1 \\ 3 \end{bmatrix}$ Let $u_3 = \begin{bmatrix} -1 \\ -2 \\ -1 \\ 3 \end{bmatrix}$.

basis

$\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ 5 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ -1 \\ 3 \end{bmatrix}$