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12. Let A be a symmetric matrix and let u and v be eigenvectors of A which belong to different eigenvalues. PROVE that $u^{\mathrm{T}}v = 0$.

Let $Au = \lambda u$ and $Av = \beta v$ with $\lambda \neq \beta$

$\lambda u^T v = \lambda v^T u = v^T A u = (Av)^T u = u^T (Av) = \beta u^T v$

u & v are column vectors

$Au = \lambda u$

$(AB)^T = B^T A^T$ and $A^T = A$ because A is symmetric

because u and Av are column vectors

because $Av = \beta v$

$\therefore (\lambda - \beta) u^T v = 0$ but both objects are numbers and $\lambda - \beta \neq 0$ so $u^T v = 0$

13. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If A and B are 2×2 matrcies with A non-singular, then the column space of AB is equal to the column space of B.

False $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ the column space of B is the x-axis

$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ non singular

$AB = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ the column space of B is the y-axis