

98  
75

Let

$$A = \begin{bmatrix} 1 & 2 & 2 & 6 & 2 & 8 \\ 1 & 2 & 3 & 9 & 2 & 8 \\ 1 & 2 & 3 & 9 & 3 & 12 \\ 2 & 4 & 5 & 15 & 5 & 20 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 3 \\ 2 \\ 4 \\ 7 \end{bmatrix}$$

6. Find a basis for the row space of  $A$ .
7. Find a basis for the column space of  $A$ .
8. Find a basis for the null space of  $A$ .
9. Solve  $Ax = b$ .

$$\left[ \begin{array}{cccccc|c} 1 & 2 & 2 & 6 & 2 & 8 & 3 \\ 1 & 2 & 3 & 9 & 2 & 8 & 2 \\ 1 & 2 & 3 & 9 & 3 & 12 & 4 \\ 2 & 4 & 5 & 15 & 5 & 20 & 7 \end{array} \right] \quad \begin{array}{l} R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow R_3 - R_1 \\ R_4 \rightarrow R_4 - 2R_1 \end{array}$$

$$\left[ \begin{array}{cccccc|c} 1 & 2 & 2 & 6 & 2 & 8 & 3 \\ 0 & 0 & 1 & 3 & 0 & 0 & -1 \\ 0 & 0 & 1 & 3 & 1 & 4 & 1 \\ 0 & 0 & 1 & 3 & 1 & 4 & 1 \end{array} \right] \quad \begin{array}{l} R_1 \rightarrow R_1 - 2R_2 \\ R_3 \rightarrow R_3 - R_2 \\ R_4 \rightarrow R_4 - R_2 \end{array}$$

$$\left[ \begin{array}{cccccc|c} 1 & 2 & 0 & 0 & 2 & 4 & 5 \\ 0 & 0 & 1 & 3 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 4 & 2 \\ 0 & 0 & 0 & 0 & 1 & 4 & 2 \end{array} \right]$$

$$\begin{array}{l} R_1 \rightarrow R_1 - 2R_3 \\ R_4 \rightarrow R_4 - R_3 \end{array} \quad \left[ \begin{array}{cccccc|c} 1 & 2 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 3 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 4 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

(6)  $[1 \ 2 \ 0 \ 0 \ 0 \ 0]$ ,  $[0 \ 0 \ 1 \ 3 \ 0 \ 0]$ ,  $[0 \ 0 \ 0 \ 0 \ 1 \ 4]$

(8) the null space is

$$\begin{array}{l} x_1 = -2x_2 \\ x_2 = x_2 \\ x_3 = -3x_4 \\ x_4 = x_4 \\ x_5 = -4x_6 \\ x_6 = x_6 \end{array}$$

basis

$$\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -4 \\ 1 \end{bmatrix}$$

(7)

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 3 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 3 \\ 3 \\ 5 \end{bmatrix}$$

(9)

$$\begin{array}{l} x_1 = 1 - 2x_2 \\ x_3 = -1 - 3x_4 \\ x_5 = 2 - 4x_6 \end{array} \quad , \quad x_2, x_4, x_6 \text{ are arbitrary}$$