54
73

PRINT Your Name:_____

There are 1~~8~~7 problems on 7 pages. Problem 1 is worth 14 points. Each of the other problems is worth 8 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible. **NO CALCULATORS.**

1. Let A be an $n \times n$ matrix. List 8 statements that are equivalent to the statement " A is nonsingular".

1) A has an inverse

2) If $AX=0$ then $X=0$

3) the columns of A are linearly independent

4) The column space of A has dimension n

5) The columns of A span $\mathbb{R}^n$

6) the rank of A is n

7) the rows of A are linearly independent

8) the row space of A has dimension n

9) $AX=b$ has a unique solution for every $b \in \mathbb{R}^n$.

10) $\lambda=0$ is not an eigenvalue of A

2. Define "linear transformation".

Let V and W be vector spaces. A <u>linear transformation</u> from V to W is a function $T: V \to W$ with

$$T(v_1 + v_2) = T(v_1) + T(v_2)$$

and $T(av_1) = aT(v_1)$

for all $v_1, v_2 \in V$ and $a \in \mathbb{R}$.