62

7. Let $\mathcal{B}$ be the basis $\begin{bmatrix}1\\-3\end{bmatrix}, \begin{bmatrix}2\\-5\end{bmatrix}$ of $\mathbb{R}^2$. Let $x = \begin{bmatrix}-2\\1\end{bmatrix}$ be a vector in $\mathbb{R}^2$.
Find the coordinate vector $[x]_{\mathcal{B}}$ of x with respect to the basis $\mathcal{B}$.

If $[x]_{\mathcal{B}} = \begin{bmatrix}c_1\\c_2\end{bmatrix}$, then $c_1\begin{bmatrix}1\\-3\end{bmatrix}+c_2\begin{bmatrix}2\\-5\end{bmatrix}=\begin{bmatrix}-2\\1\end{bmatrix}$. I solve $\left[\begin{array}{cc|c}1&2&-2\\-3&-5&1\end{array}\right] \xrightarrow{R2\mapsto R2+3R1} \left[\begin{array}{cc|c}1&2&-2\\0&1&-5\end{array}\right]$

$\xrightarrow{R1\mapsto R1-2R2} \left[\begin{array}{cc|c}1&0&8\\0&1&-5\end{array}\right]$ $\quad [x]_{\mathcal{B}} = \begin{bmatrix}8\\-5\end{bmatrix}$

check $8\begin{bmatrix}1\\-3\end{bmatrix} - 5\begin{bmatrix}2\\-5\end{bmatrix} = \begin{bmatrix}8-10\\-24+25\end{bmatrix} = \begin{bmatrix}-2\\1\end{bmatrix}$ ✓

8. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If U and V are subspaces of $\mathbb{R}^2$, then the union $U \cup V$ is also a subspace of $\mathbb{R}^2$.

False $\quad U = \left\{\begin{bmatrix}a\\0\end{bmatrix} \middle| a\in\mathbb{R}\right\}$ and $V = \left\{\begin{bmatrix}0\\a\end{bmatrix} \middle| a\in\mathbb{R}\right\}$ are subspaces of $\mathbb{R}^2$

$\begin{bmatrix}1\\0\end{bmatrix}\in U\cup V$ $\quad \begin{bmatrix}0\\1\end{bmatrix}\in U\cup V$ but $\begin{bmatrix}1\\0\end{bmatrix}+\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix}\notin U\cup V$.