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4. Suppose A is an $n \times n$ matrix and $Ax = 0$ has infinitely many solutions. Let b be a vector in $\mathbb{R}^n$.
   (a) Can $Ax = b$ have no solution? Yes
   (b) Can $Ax = b$ have exactly one solution? No
   (c) Can $Ax = b$ have infinitely many solutions? Yes
   (d) **EXPLAIN** each answer.

The hypothesis tells us that A is <u>not</u> invertible, so the IMT tells us that $AX=b$ has no solution for some choices of b. Also, if $AX=b$ has any solutions, then it has to have an infinite number of solutions because if $AX=b$ and $AY=0$ then $A(X+Y)=b$. We are told that there are infinitely many Y with $AY=0$.

5. Suppose $v_1, \ldots, v_n$ are linearly independent vectors in $\mathbb{R}^n$. Do $v_1, \ldots, v_n$ have to span $\mathbb{R}^n$? **EXPLAIN.**

Yes. The main theorem about dimension tells us that n linearly independent vectors in $\mathbb{R}^n$ are a basis for $\mathbb{R}^n$; thus, these vectors must span $\mathbb{R}^n$.

You may also give an explanation based on the IMT, see exam 3.

6. Let $\mathcal{B}$ be the basis $\begin{bmatrix} 3 \\ -5 \end{bmatrix}, \begin{bmatrix} -4 \\ 6 \end{bmatrix}$ of $\mathbb{R}^2$. Suppose that x is the vector in $\mathbb{R}^2$ whose coordinate vector with respect to the basis $\mathcal{B}$ is $[x]_{\mathcal{B}} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$. What is the usual representation of x?

$$X = 5\begin{bmatrix} 3 \\ -5 \end{bmatrix} + 3\begin{bmatrix} -4 \\ 6 \end{bmatrix} = \begin{bmatrix} 15-12 \\ -25+18 \end{bmatrix} = \begin{bmatrix} 3 \\ -7 \end{bmatrix}$$