7. Let $\mathcal{B}$ be the basis $v_1 = \begin{bmatrix} 1 \\ 8 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ -9 \end{bmatrix}$ of $\mathbb{R}^2$ and let x be the vector $x = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$. Find the coordinate vector $[x]_{\mathcal{B}}$ of x with respect to the basis $\mathcal{B}$.

$c_1v_1 + c_2v_2 = x$. Find c_1 & c_2

$$\left[\begin{array}{cc|c} 1 & 2 & 4 \\ 8 & -9 & 7 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & -27 & -27 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 1 \end{array}\right]$$

$c_1 = 2$
$c_2 = 1$ ✓ $\quad 2\begin{bmatrix} 1 \\ 8 \end{bmatrix} + 1\begin{bmatrix} 2 \\ -9 \end{bmatrix} = \begin{bmatrix} 4 \\ 16-9 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$ ✓

$\begin{bmatrix} 2 \\ 1 \end{bmatrix} = [x]_{\mathcal{B}}$

8. Suppose a nonhomogeneous system of nine linear equationsin ten unknowns has a solution for all possible constants on the right side of the equations. Is it possible to find two nonzero solutions of the associated homogeneous system that are not multiples of each other? Explain.

Consider A which is 9×10. We are told $Ax = b$ has a solution for all $b \in \mathbb{R}^9$

So $\dim \text{col space } A = 9$ $\quad \therefore \dim \text{null space } A = \#\text{Cols} - 9 = 10 - 9 = 1$.

$\therefore$ Every vector in the null space of A is the multiple of a fixed vector.

The answer is no.