4. Suppose that A is a square matrix and A^2 is the identity matrix. What are the possible eigenvalues for A? Prove your answer.

If λ is an evalue for A, then there exists a vector X with $AX = \lambda X$

So $A^2X = \lambda AX = \lambda^2 X$. But $A^2 =$ identity so $X = \lambda^2 X$ or $(1-\lambda^2)X = 0$

a number a non-zero vector

$\therefore\ 1-\lambda^2 = 0$ so $1 = \lambda^2$ so $\lambda = +1$ or -1

5. Give an example of a 3×3 matrix A whose eigenvalues are 0 and 1 such that the eigenspace of A which belongs to $\lambda = 0$ has dimension 1 and the eigenspace of A which belongs to $\lambda = 1$ also has dimension one. The matrix A is to have no other eigenvalues other than 0 and 1.

$A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

The e space belonging to 1 has basis $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. The e.space belonging to 0 has basis $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

6. Let V be the set of all 3×3 skew-symmetric matrices. (The matrix A is skew-symmetric if $A^{\mathrm{T}} = -A$.) The set V is a vector space. You do NOT have to prove this. Give a basis for V. NO justification is needed.

$\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$